

The Number Machine

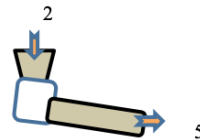
Problem wording

Today we're going to play a game. This machine changes numbers. It's your job to figure out how it works.

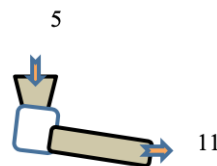
Identification of specific cases and recognition of structure

The teacher introduces the problem to the class as a whole, making sure students understand what the machine does. The use of pictures is recommended to support the explanation. Examples should be given one by one for students to try to determine the number they think the machine will deliver.

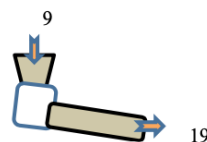
- If we put in a 2 what number do you think will come out? It's 5 [which had been hidden].



- If we put in a 5 what number do you think will come out? It's 11 [which had been hidden].

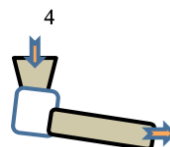


- If we put in a 9 what number do you think will come out? It's 19 [which had been hidden].

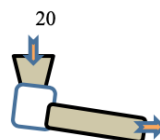


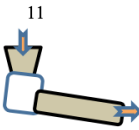
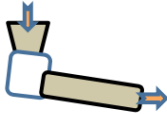
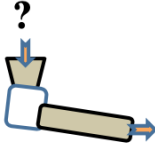
Students are then asked to explore other specific cases, individually or in small groups.

1. What number comes out if we put a 4 in?



2. What number comes out if we put a 20 in?



	<p>3. What number comes out if we put an 11 in?</p>  <p>4. Put whatever number you choose in. What number will come out?</p>  <p>Formulating a conjecture</p> <p>5. Explain how the machine works. 6. If ‘?’ is a number we don’t know, how would you explain what number comes out?</p>  <p>Validating a conjecture</p> <p>Mark the following statements as true or false.</p> <p>7. If we put in a 6, the number that comes out is 12. 8. If we put in a 13, the number that comes out is 27.</p> <p>Generalising a conjecture</p> <p>9. If I put A in the machine, C comes out. 10. If I put A in the machine, $2xA+1$ comes out. 11. If I put A in the machine, $A+A$ comes out. 12. The number that comes out is always the number that goes in plus 3. 13. The number that comes out is always triple the number that goes in. 14. The number that comes out is always twice the number that goes in plus 1.</p>
<p>Purpose</p>	<ul style="list-style-type: none"> • To build on specific cases to discover the rule governing the function. • To apply the rule governing the function to specific numerical cases. • To generalise the functional relationship. • To generalise the functional relationship in cases involving an indeterminate quantity.

<p>Suggestions for classroom delivery</p>	<p>If students are unable to grasp how the machine works, the numbers can be represented pictorially to enable them to compare the similarities and differences between input and output. For instance:</p> <div data-bbox="826 427 1082 600" data-label="Diagram"> <p>The diagram shows a simple machine with a hopper on the left and a chute on the right. Five green dots are shown entering the hopper from above. An arrow points from the hopper into the machine. From the chute, seven green dots are shown exiting to the right. This illustrates a transformation where the input is 5 and the output is 7.</p> </div> <p>In this task, the functional relationship has to be determined from the initial examples discussed with the class as a whole, given that the wording gives no clue. The teacher should consequently analyse the first three cases with students and if they fail to understand, suggest other pairs of inputs and outputs, such as: (3,7), (10, 21) and (1, 3).</p> <p>The problem poses the indeterminate quantity issue in three ways: with a question mark ?, natural language and letters. The teacher should establish a relationship among them as far as possible and allow students to express their ideas with the type of representation they feel most comfortable with.</p>
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